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A Note on Dynamics in the Modal Interpretation

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I summarize recent results about the construction of a dynamics in the modal interpretation. Some properties of the resulting dynamics are reviewed.

1. INTRODUCTION

In this brief note, I wish to summarize the content of my contribution to the IQSA meeting in Berlin. These results are published in fuller form elsewhere (Bacciagaluppi and Dickson, 1997; Bacciagaluppi *et al.*, n.d.; Bacciagaluppi, 1998). (The last paper is mainly a review article, covering most of the work done on this topic.)

I treat the question of finding a dynamics for the modal interpretation in the version of Kochen (1985), Dieks (1988, 1989), and Healey (1989), as generalized by Vermaas and Dieks (1995). In this interpretation, the spectral decomposition of the reduced state ρ of a system plays a privileged role, in the sense that the complete state of the system is considered to be a pair (ρ, P_i) in which P_i is one of the eigenprojections of the reduced state ρ, and it is postulated to occur with probability $\mathrm{Tr}(\rho P_i)$ when the state is ρ. The question thus arises of formulating a (stochastic) dynamics for the evolution of the possessed properties (hidden variables) P_i. This dynamics will have to respect also the single-time distributions for different systems $\mathcal{H}^1 \otimes \ldots \otimes \mathcal{H}^N$,

$$p(P^1_{i_1}, \ldots, P^N_{i_N}) := \mathrm{Tr}(\rho P^1_{i_1} \otimes \cdots \otimes P^N_{i_N}) \tag{1}$$

characteristic of the Vermaas–Dieks (1995) version.

The construction of such a dynamics is obtained by generalizing the dynamics sketched by Bell (1984) and more fully discussed by Vink (1993),

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which is a stochastic version of the dynamics of the Bohm (1952) theory, in which, however, not position, but some *discrete* observable is given the privileged status of a *beable.*

In the context of the modal interpretation, the analogy with the Bohm theory has been stressed by Bub (1992, 1994), who has set up a framework for modal interpretations understood as Bohm-like theories with a discrete privileged observable R instead of the positions of the particles. [The definitive paper on this framework is Bub and Clifton (1996).] Bub (1995) has also adopted Bell's (1984) dynamics as developed by Vink (1993). Within Bub's framework, we can fit the Vermaas–Dieks (1995) version of the modal interpretation by taking not the positions of the particles to be privileged, but the observables with the eigenprojections $P^j_{i_j}$ (i.e., the reduced states ρ^j of the particles), so that the products of the $P^j_{i_j}$ define the eigenprojections [in the following simply denoted as $P_i(t)$] of a *time-dependent* privileged observable $R(t)$.

Implicit in the above are the following two ideas. (A) The assignment of properties to a system via the spectral decomposition of its reduced state is not applied to composite systems, since the resulting properties would generally not commute with the properties of the component systems, and this would make it impossible to define a privileged observable $R(t)$. This seems to be required anyway by a series of recent no-go theorems directed against the possibility of simultaneously assigning properties also to other systems (Bacciagaluppi, 1995; Clifton, 1996; Vermaas, 1997). See also the review by Bacciagaluppi and Vermaas (1998), and the discussion of the resulting theory by Dieks (1998).

(B) The $P_i(t)$ at different times must somehow be identifiable as the *same* P_i that have evolved over time. The need for the existence of such trajectories will become clearer in the actual construction of the dynamics [where, in fact, the trajectories $P_i(t)$ need to be differentiable]. The question of the time evolution of the $P_i(t)$ has been treated by Bacciagaluppi *et al.* (1995), who, using quite standard theorems of perturbation theory, identify conditions under which the $P_i(t)$ form even *analytic* trajectories (or more precisely, trajectories that possess analytic continuations). In the following, we shall assume that such conditions hold. In fact, although we shall not go into details, we shall assume the strongest such condition, namely that the *total* Hilbert space be finite-dimensional, which also simplifies the discussion in other respects.

2. CONSTRUCTION

A sketch of the dynamics can be given as follows. In the case of a discrete Markov process (and if we consider large enough and thus closed systems, the assumption of the Markov property will be warranted), one can reconstruct the (multitime) distribution functions of the process from its (two-time) transition functions $p_{ji}(t, s)$, representing the transition probability from the state i at time s to the state j at time $t > s$. Further, one can in turn recover the transition functions from the so-called infinitesimal transition probabilities or growth rates $t_{ji}(t)$ by standard techniques (Kolmogorov, 1931; Feller, 1940). One can thus specify a dynamics in terms of the growth rates $t_{ji}(t)$.

In the Bell–Vink dynamics one sets

$$t_{ji}(t) := \max\left\{0, \frac{j_{ji}(t)}{p_i(t)}\right\} \tag{2}$$

where, assuming that the state of the total system is $\langle\Psi(t)\rangle$, $p_i(t)$ is the single-time distribution

$$p_i(t) := \langle\Psi(t)|P_i|\Psi(t)\rangle \tag{3}$$

and $j_{ji}(t)$ is a probability current, given by

$$j_{ji}(t) := 2\,\mathrm{Im}[\langle\Psi(t)|P_j H P_i|\Psi(t)\rangle] \tag{4}$$

From the Schrödinger equation, one sees that (4) satisfies the following continuity equation:

$$\dot{p}_j(t) = \sum_i j_{ji}(t) \tag{5}$$

Vink (1993) has shown that this dynamics reduces to that of the Bohm theory in the appropriate continuum limit. However, there are many more dynamics that are consistent with the given single-time distributions (3). Instead of (2) one can take any solution of the equation

$$j_{ji}(t) := t_{ji}(t)p_i(t) - t_{ij}(t)p_j(t) \tag{6}$$

and for $j_{ji}(t)$ any solution of the continuity equation (5) that is antisymmetric in j and i [cf. (6)].

The generalization to the time-dependent case (Bacciagaluppi and Dickson, 1997), in which $P_i(t)$ becomes time-dependent and

$$p_j(t) = \langle\Psi(t)|P_j(t)|\Psi(t)\rangle \tag{7}$$

is obtained by adding an appropriate term to the current (4):

$$\begin{aligned} j_{ji}(t) = {} & 2\,\mathrm{Im}[\langle\Psi(t)|P_j(t)HP_i(t)|\Psi(t)\rangle] \\ & + \langle\Psi(t)|P_j(t)\dot{P}_i(t) - \dot{P}_i(t)P_j(t)|\Psi(t)\rangle \end{aligned} \tag{8}$$

[The choice of the extra term is highly nonunique, but is argued for by Bacciagaluppi and Dickson (1997) and Bacciagaluppi (1998).] Using again (2), this current yields infinitesimal transition probabilities and thus the desired generalization of the dynamics to the time-dependent case.

It should be noticed, however, that as discussed by Bacciagaluppi and Dickson (1997) and Bacciagaluppi (1998), whenever the $p_i(t)$ have a zero, the $t_{ji}(t)$ become singular, by (2). (Indeed, one can show that they have a nonintegrable singularity.) As a consequence, the regularity assumptions for the existence and uniqueness of solutions to the Kolmogorov equations (Feller, 1940) are violated. The situation is quite similar to that of the Bohm theory, in which the guidance equation will also become singular when the wavefunction is zero. In the Bohm theory it has been shown that the guidance equation admits unique solutions for wide classes of potentials and generic initial conditions (Berndl *et al.*, 1995). It is plausible that similar results can be obtained also in the case of the (generalized) Bell–Vink dynamics.

3. PROPERTIES OF THE DYNAMICS

I shall conclude by mentioning a few properties of the dynamics sketched above (which also give a partial justification for the particular choice of a solution), with particular reference to the analogy with the Bohm theory.

First of all, one can show that whenever the possessed properties of one particle commute with the interaction Hamiltonian during an interval $[s, t]$, then the possessed properties of that particle are *stable*, in the sense that they undergo no stochastic transitions in that time interval (Bacciagaluppi and Dickson, 1997; Bacciagaluppi, 1998).

This is a generalization of the analogous result for freely evolving systems derived by Vermaas (1996). As a consequence, one can rederive some further results by Vermaas (1996) for special cases of interacting systems (cases in which at one time the properties of one particle are perfectly correlated with the properties of some other, freely evolving, particle). In particular, one can then easily show by example that: (a) two-time transition probabilities by no means always have the form of the transition probabilities of the projection postulate (although in some cases they do); (b) the evolution of open systems is in general non-Markovian.

The stability result just mentioned can also be used to analyze properties of the dynamics in toy models of measurement, as done by Bacciagaluppi and Hemmo (1998) and by Dickson and Clifton (1998). The former explain the mechanism of state preparation in the modal interpretation for such models, and analyze how in the models the modal interpretation behaves like a stochastic hidden variables theory. The latter show that the probabilities in any dynamics that exhibits stability (in the above sense) must violate fundamental Lorentz invariance, in the sense that they will be frame dependent. One obtains frame dependence of actual trajectories of possessed properties as in the Bohm theory. Dickson and Clifton (1998) also discuss the emergence of Lorentz invariance at the phenomenological level.

Finally, the existence of a dynamics that explains how the probability distribution over the definite properties at time t is mapped to the probability distribution at time t' clarifies the status of these single-time distributions as purely epistemic, again as in the Bohm theory. However, while in the Bohm theory, which is deterministic, a 'correct' initial distribution may perhaps need to be postulated, in the stochastic dynamics sketched above it can be derived, in the sense that it can be shown that for any distribution $d_i(t)$ other than (7),

$$\frac{d}{dt}\sum_i |d_i(t) - p_i(t)| \leq 0 \tag{9}$$

Whether this leads, indeed, to convergence of the two distributions and how quickly depends further on the presence of sufficient interactions (Bacciagaluppi *et al.*, n.d.). But this result can be interpreted as indicating that an initial distribution need not be introduced in the theory as a postulational element.

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